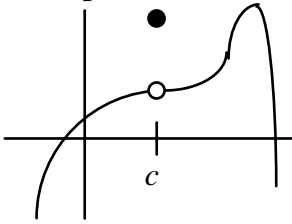


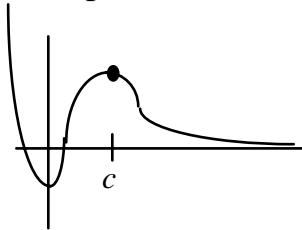
Limits

Note: The limit of a function at a value of x has nothing to do with what the function equals at x . ($\lim_{x \rightarrow c} f(x)$ doesn't necessarily equal $f(c)$.)

example 1



example 2



example 3

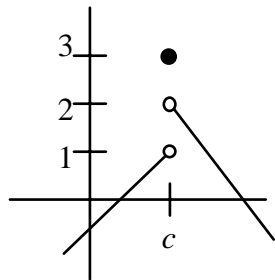
$$f(x) = \begin{cases} \frac{x^2 + x}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

Sometimes a function doesn't have a limit at a particular point. Example 4

$$f(x) = \frac{1}{x}.$$

Some functions approach a different value as x approaches a constant from the left (the negative (-)) than as x approaches that constant from the right (the positive (+)).

example 5



example 6

$$f(x) = \begin{cases} 2x^2 & \text{when } x > 1 \\ 2x - 1 & \text{when } x \leq 1 \end{cases}$$

If $\lim_{x \rightarrow c^+} f(x) \neq \lim_{x \rightarrow c^-} f(x)$ we say $\lim_{x \rightarrow c} f(x)$ does not exist.

When a function grows or shrinks without bound we say it goes towards plus or minus infinity. In example 4: $\lim_{x \rightarrow 0^+} f(x) =$ and

$\lim_{x \rightarrow 0^-} f(x) =$, even though $\lim_{x \rightarrow 0} f(x)$ *d.n.e.* (does not exist).

We can also evaluate the limit of a function as x approaches $\pm \infty$.

In example 2: $\lim_{x \rightarrow \infty} f(x) = 0$ and $\lim_{x \rightarrow -\infty} f(x) = \infty$

In example 4: $\lim_{x \rightarrow \infty} f(x) = 0$ and $\lim_{x \rightarrow -\infty} f(x) = 0$