Math 151 Test 4 March 26, 1998 Name _____

Show all work which leads to an answer. Include the equations and graphs anything that you used a graphing calculator to solve.

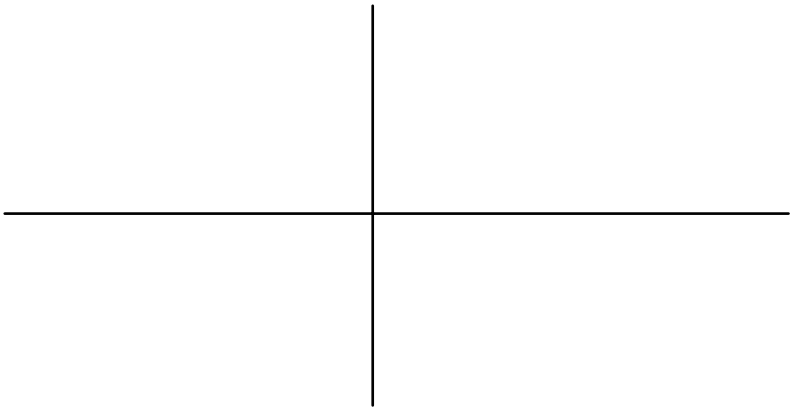
Do not attach scrap paper. Instead, transfer your work onto this test.

1. Sketch the graph of a continuous function *f* which meets following conditions: (9 pts)

$$f(0)=4;\quad f(3)=f(-3)=2;\quad f'(0)=0;\quad f'(x)>0 \text{ if } x<0;$$

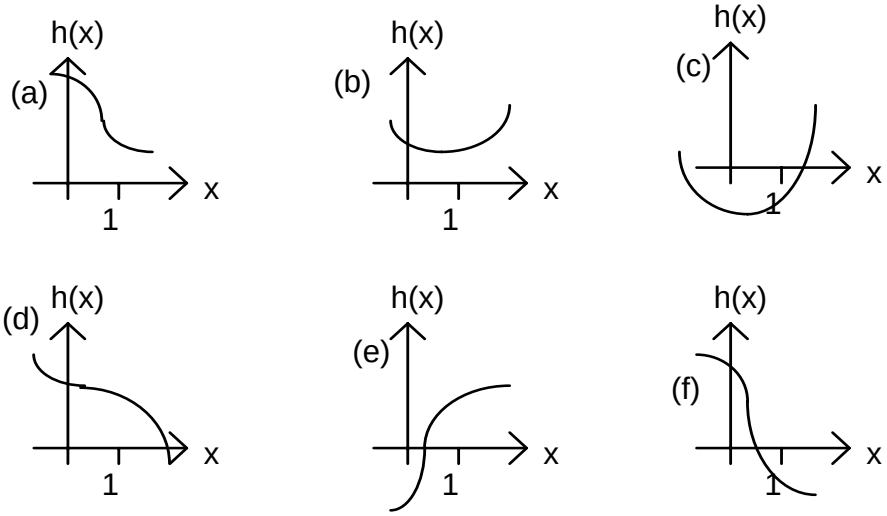
$$f'(x)<0 \text{ if } x>0;\quad f''(x)<0 \text{ if } |x|<3;\quad f''(x)>0 \text{ if } |x|>3;$$

$$\lim_{x\to\infty} f(x)=-2;\quad \lim_{x\to-\infty} f(x)=-2$$



2. Which of the following graphs match the given information? _____

$h(1) > 0$, $h'(1) > 0$, $h''(1) < 0$ (6 pts)

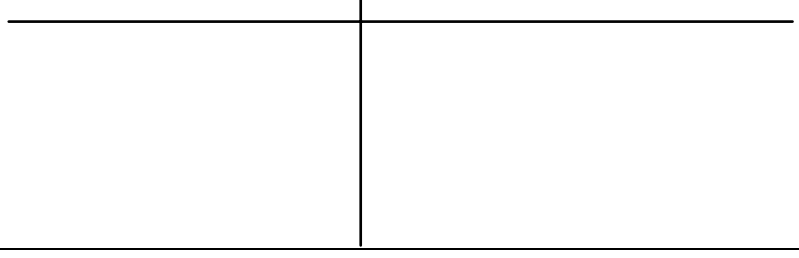


3. The graph of the function $f(x) = ax^5 + bx^2 + c$ contains a critical point at (0, 2) and an inflection point at (-1, 6) . Find *a*, *b*, and *c*. (10 pts)

4. Give the dimensions of an open-topped cylindrical container of volume 5 cubic meters which requires the least amount of material to be used. Assume the thickness for the bottom and sides is the same. (10 pts)

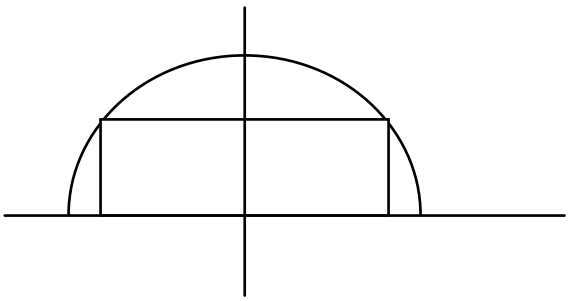
5. Let $y = \dfrac{3x}{(x+8)^2}$.

a.) Find the x and y intercepts.(2 pts)	b.) Find the equation of all linear asymptotes. (2 pts)
c.) Find the first and second derivatives (on back, then transfer your factored answers to this space) . (6 pts)	d.) Find all critical point and inflection the points (both coordinates). (4 pts)
e.) Use a sign cart to show(or state in interval notation) where the first and second derivatives are positive and negative. (4 pts)	f.) Sketch the graph below. **DO NOT DRAW TO SCALE**, instead show all important details). Make sure it is easy for me to tell where the graph is increasing, decreasing, concave up, and concave down. Draw the graph as a solid line and draw the asymptotes as dotted lines. (2 pts)



6. A rectangle is to be inscribed in a half circle of fixed radius *b* with its base on the flat side of the half circle. Find the dimensions of the rectangle with the largest area.

(9 pts)



7. For each of the following, determine whether the function *f* satisfies the hypothesis of the Mean Value Theorem on the indicated interval [a, b] and if so, find all values *c* in (*a*, *b*) such that $\frac{f(b)-f(a)}{b-a} = f'(c)$.

(12 pts)

a.) $f(x) = 5x^2 - 3x + 1$, [1, 3]

b.) $f(x) = x + \frac{8}{x}$, [1, 4]

c.) $f(x) = 2x + 3x^{2/3}$, [-8, 27]

8. Find the general antiderivative for each of the following: (15 pts)

a.) $x^{-3} + 2\sqrt{x}$

b.) $\csc^2 x + \sin 5x$

c.) $x+2f(x)$ if the antiderivative of $f(x)$ is $x^2 + 3$

9. Find the equation for a particles position at time *t sec*, if its acceleration is $8t$ *ft*/*sec*^{2}, its initial velocity is 5 *ft*/*sec*, and its initial position is at 7 *ft*. (9 pts)