Math 151 Test 5 April 13, 1998 Name _____

Show all work needed to complete a problem on these pages.

(5 pts.) 1. Which one of the following limits represents: $\int_{-2}^{5} 5x^3\,dx$ using the right hand endpoints of a Riemann Sum.

(a.) $\lim_{n\to\infty} \sum_{i=1}^{n} 5\left(\frac{7i}{n}\right)^3\left(\frac{7}{n}\right)$

(b.) $\lim_{n\to\infty} \sum_{i=1}^{n} 5\left(\frac{7i}{n}-2\right)^3\left(\frac{1}{n}\right)$

(c.) $\lim_{n\to\infty} \sum_{i=1}^{n} 5\left(\frac{i}{n}\right)^3\left(\frac{7}{n}\right)$

(d.) $\lim_{n\to\infty} \sum_{i=1}^{n} 5\left(\frac{7i}{n}-2\right)^3\left(\frac{7}{n}\right)$

(e.) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i}{n}\right)^3\left(\frac{1}{n}\right)$

(12 pts.) 2. Suppose *f* and *g* are continuous and that $\int_3^7 f(x)dx = 8$, $\int_5^7 f(x)dx = 10$, and $\int_3^7 g(x)dx = 2$. Evaluate:

(a.) $\int_7^5 f(x)dx$

(b.) $\int_3^5 f(x)dx$

(c.) $\int_3^7 \left[f(x) + 2g(x)\right]dx$

(d.) $\int_7^7 f(x)dx$

3. Recall that Simpson's Rule to approximate $\int_a^b f(x)dx$ is $S = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right)$ where the interval [*a*, *b*] is partitioned into *n* equal increments (*n* even) of width *h* and each $y_i = f(x_i)$. The error E_s is given by: $|E_s| \le \frac{b-a}{180} h^4 M$ where *M* is any upper bound on the values of $\left|f^{(4)}\right|$ on the interval [*a*, *b*].

(6 pts.) (a.) Use the error formula to find the smallest integer value of n needed to guarantee accuracy of 10^{-4} using Simpson's Rule for the following definite integral:

$$\int_1^3 \frac{1}{48x}\,dx$$

(4 pts.) (b.) Use your answer to part (a.) to calculate the above definite integral with accuracy to the ten-thousandths place.

(15 pts.) 4. Determine whether the following integrals is correct. Show work to justify your answer.

(a.) $\int x\cos x = \frac{1}{2}x^2 \sin x + C$

Correct?: yes or no
(circle one)

(b.) $\int x\cos x = \frac{1}{2}x^2 \cos x + x\sin x + C$

Correct?: yes or no
(circle one)

(c.) $\int x\cos x = x\sin x + \cos x + C$

Correct?: yes or no
(circle one)

(8 pts.) 5. Find the total area enclosed by the x-axis, the lines $x = 1$, $x = 4$, and the curve $y = x^2 - 7x + 10$.

(6 pts.) 6. Evaluate: $\dfrac{d}{dx}\displaystyle\int_2^{x^5} \left(t^2 + 3t + 1\right)^3 dt$

(8 pts.) 7. Solve the following initial value problem. Write your answer in terms of an integral.

$\dfrac{dy}{dx} = \sqrt{x^4 + 1}$, $y = 1$ when $x = \pi$

(8 pts.) 8. Solve the following initial value problem:

$$\frac{d^2y}{dx^2} = \cos x + 2 \quad ; \quad \frac{dy}{dx} = 2 \text{ and } y = 0 \text{ when } x = 0.$$

(9-12) Evaluate the following without a calculator:

(7 pts.) 9. $\int \csc 2x \cot 2x \, dx$

(7 pts.) 10. $\int_1^4 \frac{x^2 + 1}{\sqrt{x}} dx$

(7 pts.) 11. $\int ax^2\left(bx^3+c\right)^5\,dx$ Assume *a*, *b*, and *c* are constants.

(7 pts.) 12. $\int_0^{\pi/9} \sec^2 3x \tan 3x\, dx$