Math 151 Test 6 April 28, 1998 Name _____

Show **all of your work** on these pages. Transfer all relevant scratch work from your scrap paper to this test.

1. The earth's atmospheric pressure is modeled by assuming that the rate of change, $\frac{dp}{dh}$ of pressure p with respect to altitude h (above sea level) is proportional to p. Suppose that the pressure at sea level is 1000 millibars and that the pressure at an altitude of 25 km is 85 millibars.

(6 pts) a. Find the equation for $p(h)$ using the fact that $\frac{dp}{dh} = kp$ where k is a constant, $p = p_o$ when $h = 0$. Use the values above to determine p_o and k.

(4 pts) b. What is the atmospheric pressure at $h = 50$ km?

(4 pts) c. At what altitude does the pressure equal 900 millibars?

2. Evaluate the following integrals **exactly** without a calculator. **Simplify your answer completely.**

(9 pts) a. $\int_0^{\sqrt{\ln 2}} xe^{3x^2}\,dx$

(9 pts) b. $\int_1^8 \frac{3^{\log_2 x}}{x}\,dx$

(9 pts) 3. Solve the following initial value problem:

$$\frac{dy}{dx} = 3 + \frac{1}{2x}, \quad x > 0 \qquad \text{where } y = 2 \text{ when } x = 1.$$

(10 pts) 4. Differentiate $y = 5^{3x} + \ln\left(\frac{3x^2}{2^x + 1}\right)$ (You do not need to simplify.)

(9 pts) 5. Use logarithmic differentiation to find and solve for $\frac{dy}{dx}$ in terms of x:

$$y = \frac{(x + 2)(x^2 - 1)^2}{\sqrt{x + 3}}$$

(10 pts) 6. Find the **exact** equation of the tangent line to the curve $y = (x+2)^{x-1}$ at $x = 3$. (Expect a messy answer.)

7. Let $f(x) = x^3 + x + 1$.

(6 pts.) a. Calculate $f'(x)$ and use it to show that f is one-to-one, thus showing f has an inverse function.

(4 pts.) b. Fill in the following charts:

x	$f(x)$
0	
1	
2	
3	

x	$f^{-1}(x)$
	0
	-1
	-2
	-3

(6 pts.) c. Evaluate $\frac{df^{-1}}{dx}$ at $(3, f^{-1}(3))$

(6 pts) 8. Evaluate:

a. $\lim_{x\to\infty} 0.9999^{x}$

b. $\lim_{x\to-\infty} 0.9999^{x}$

(8 pts) 9. Find the absolute maximum and minimum value of $y = \sqrt[3]{x}\, e^{x}$ on [-1, 0].